

From Filter Quotient Model Categories to Type Theory

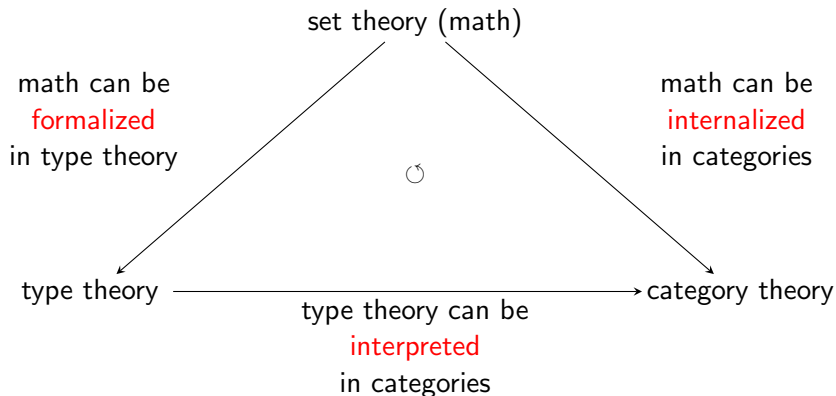
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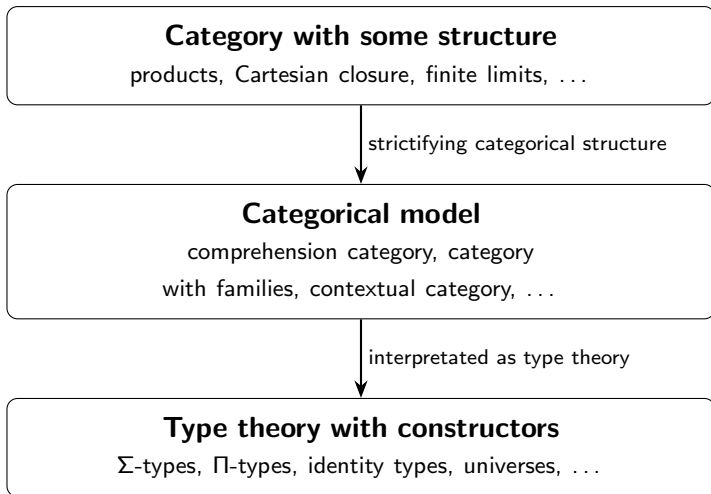
17.07.2026

Shulman's trinitarianism¹

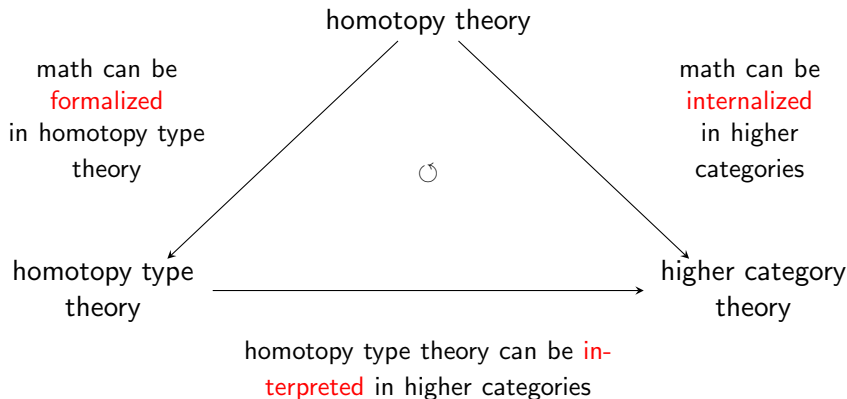


¹<https://home.sandiego.edu/~shulman/papers/trinity.pdf>

Models of Type Theory via Category Theory



Shulman's homotopical trinitarianism²



²<https://home.sandiego.edu/~shulman/papers/trinity.pdf>

Models of Homotopy Type Theory via ∞ -Category Theory

∞ -category with some structure

finite products, Cartesian closure, finite limits, locally Cartesian closure, ...

↓ strictifying homotopical structure

Model category with some structure

simplicial model category, combinatorial model category, model topos, ...

↓ strictifying categorical structure

Categorical model via Fibration Category

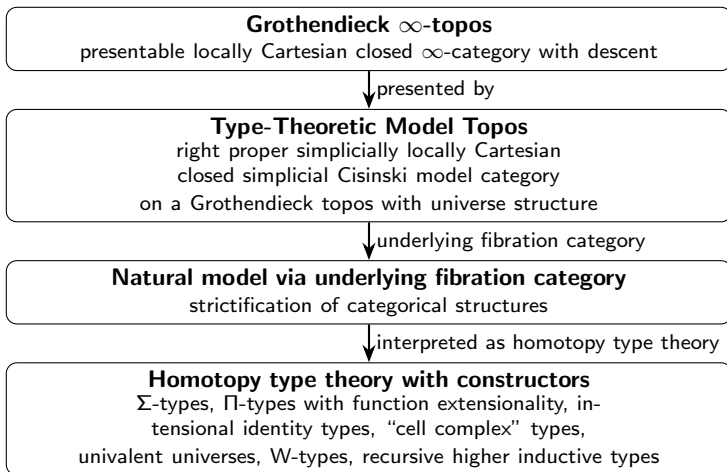
comprehension category, contextual category, natural model ...

↓ interpreted as type theory

Homotopy type theory with constructors

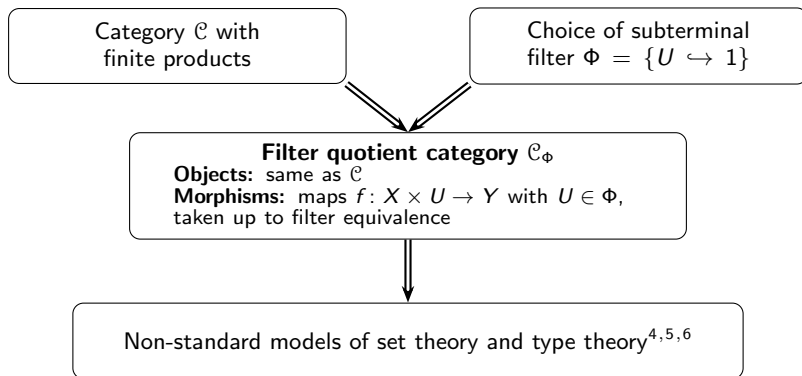
Π -types, intensional identity types, univalent universes, higher inductive types, ...

From ∞ -Topoi to Homotopy Type Theory³



³M. Shulman, *All $(\infty, 1)$ -Topoi have strict univalent universes*, arXiv:1904.07004, 2019.

Filter Quotient Construction

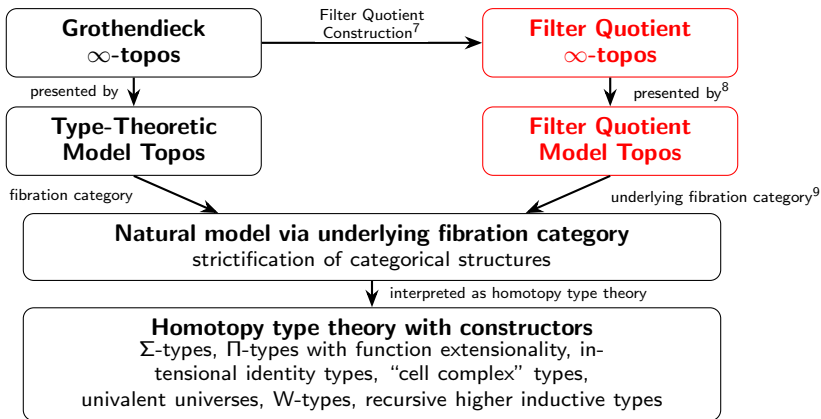


⁴M. Tierney, *Sheaf theory and the continuum hypothesis*, Lect. Notes in Math. 274, Springer, 1972, 13–42.

⁵H. Volger, *Ultrafilters, ultrapowers and finiteness in a topos*, JPAA 6(3), 1975;

⁶M. Adelman and P. T. Johnstone, *Serre classes for toposes*, BAMS 25(1), 103–115, 1982.

From Filter Quotient ∞ -Categories to HoTT

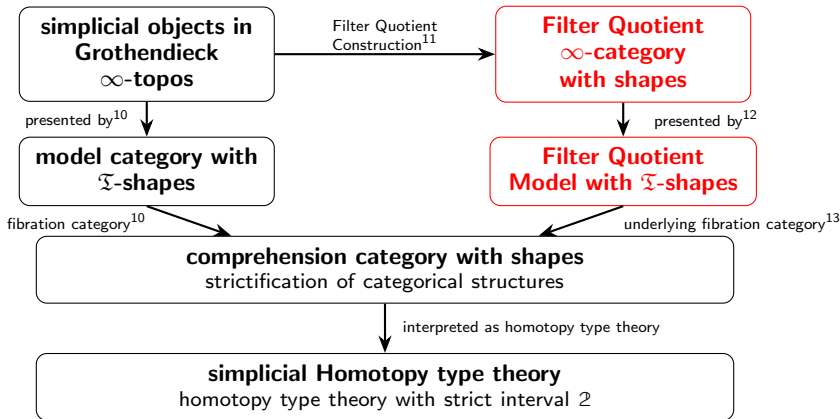


⁷ N. Rasekh, *Filter quotients and non-presentable $(\infty, 1)$ -toposes*, JPAA 225(12), 106770, 2021.

⁸ N. Rasekh, *Filter Quotient Model Structures*, arXiv:2508.07735, 2025.

⁹ N. Rasekh, *Non-Standard Models of Homotopy Type Theory*, arXiv:2508.07736, 2025.

From Filter Quotient ∞ -Categories to sHoTT



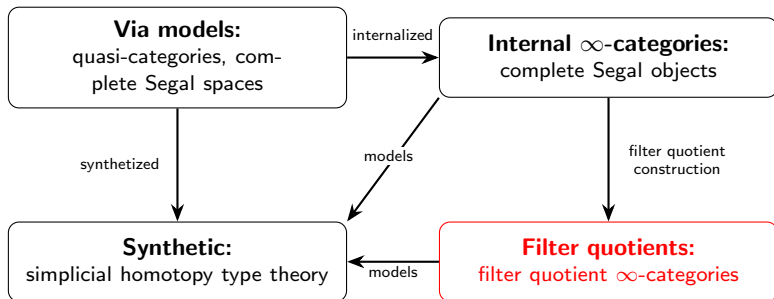
¹⁰ E. Riehl and M. Shulman, *A type theory for synthetic ∞ -categories*, Higher Structures 1(1), 147–224, 2017.

¹¹ N. Rasekh, *Filter quotients and non-presentable $(\infty, 1)$ -toposes*, JPAA 225(12), 106770, 2021.

¹² N. Rasekh, *Filter Quotient Model Structures*, arXiv:2508.07735, 2025.

¹³ N. Rasekh, *Simplicial Homotopy Type Theory is not just Simplicial*, arXiv:2508.07737, 2025.

What is an ∞ -category?



The End!

If you have any questions/ideas/feedback, talk to me:

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- Website: <https://nimarasekh.github.io/>
- Zulip: [Zulip Link](#)